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Analytical solutions by the hodograph method to hydrodynamic problems for oil and gas traps

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Abstract

Resting gas, oil and combined traps in equilibrium with moving water are studied by the methods of complex analysis (conformal mapping of hodograph circular polygons onto the strip in the complex potential plane). For a monocline dipping at an arbitrary angle and a one-phase trap two regimes (diffuser and confuser) are possible and explicit analytical solutions for the shape of a sharp interface between water and hydrocarbon are derived in terms of hypergeometric functions. The free surface is shown to coincide with the Saffman–Taylor finger in the Hele–Shaw apparatus if the monocline axis is vertical. For a gas–oil trap the interface consists of two branches along which the isobaric condition (constant pressure in the gas aloft) and the condition of a linear increase of pressure with depth (in the oil phase separated by a horizontal hydrostatic interface from the gas finger and by a hydrodynamic interface from water) are reduced to a standard hodograph representation through two touching circles. A critical regime is analyzed when the free surface does not have an inflexion point.

Keywords: Petroleum traps; Aquifer; Sharp interface; Complex potential; Hodograph; Analytic functions

1. Introduction

As oil deposits are depleting the attention of petroleum geologists focuses on hydrocarbon reservoirs of smaller sizes and/or of non-trivial locations that calls for more subtle methods of prospecting and further exploitation. One of important modes of hydrocarbon retention in geometrically non-trivial patterns is trapping, which structural and stratigraphic features are thoroughly examined and reported elsewhere (Dalberg, 1982; Jenyon, 1990).

Moving water is one of the main carriers of hydrocarbons (Levorsen, 1967). Hence, hydrogeological conditions in general and the patterns of water–hydrocarbon interaction in particular are crucial for understanding the mechanisms of entrapment and accumulation of large bodies of gas and oil (Magara, 1986). Flows of dispersed oil 'bubbles' in water within a pore channel or the encumbrance of water movement by floating hydrocarbon ganglia on the scale of an aquifer, side-by-side movement of several continua in separated pores of different sizes or within

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